Physics paper VI
Group A: Statistical Physics

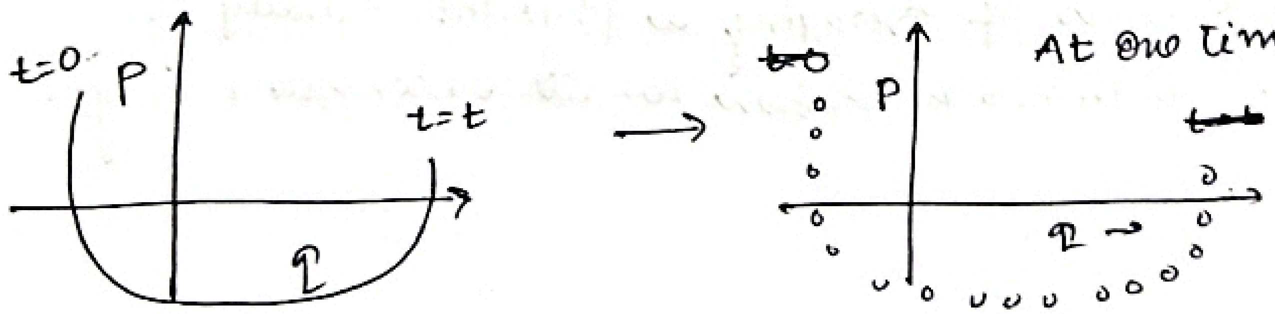
Ensemble

For a large number of particles in the system, it is difficult to find equation of motion and solve them to calculate the time average of a physical quantity of interest.

Concept of Ensemble was introduced by Gibbs in 1900 for calculating average value of the physical quantity.

Each point in a phase develops out of the previous one in time. Hence, with the passage of time, a system goes through various microstates (phase points) though the macrostate of the system remains unchanged.

The time dependent picture is replaced by static one in which all the phase points are supposed to exist simultaneously at one time.



A collection of these simultaneously existing imaginary systems is known as an ensemble.
In the ensemble, the elements are replica of the actual system of interest where microstate differs from one to another, but the macrostate remains unchanged.

N V E	N V E	N V E	N V E
N V E	N V E	N V E	N V E
N V E	N V E	N V E	N V E
N V E	N V E	N V E	N V E

In the above figure collection of simultaneously existing Gibbs points, which corresponds to different microstates of the system is shown, which is also known as ensemble.

Macrostate of all these elements of the ensemble is the same defined by N, V and E.

~~Now~~ Now, instead of taking time average of a physical quantity of the system which is evolving with time we can take average over an ~~es~~ ensemble of the system where the elements are existing at one time simultaneously. This process of averaging a physical quantity over the ensemble is known as the ensemble average.

Ensemble Average

For calculating the ensemble average of a physical quantity, Let us consider a physical system comprising a large number N of identical particles which are moving freely from one place to another within the system.

Ensemble average of a physical quantity is expressed in terms of a density function $\rho'(q, P, t)$.

This function ρ' represents the number density of phase points in the phase space.

The number of phase points in the volume element ~~is given by~~

$$dq\,dp = dq_1\,dq_2 \cdots dq_{3N}\,dp_1\,dp_2 \cdots dp_{3N}$$

around a point (q, P) in the phase space is given by $\rho'(q, P, t)\,dq\,dp$.

Total number of phase points in the phase space = Total number of the elements in the ensemble of the system (M)

$$\int \rho'(P, q, t)\,dq\,dp = M$$

The normalized density function, denoted by $\rho(q, P, t)$ and called as the probability density is

$$\boxed{\rho(q, P, t) = \frac{\rho'(q, P, t)}{M}}$$

The probability density satisfies the normalization condition

$$\int \rho(q,p,t)\, dq\, dp = 1$$

The function $\rho(q,p,t)$ is also called as the statistical distribution function.

The probability (dw) of finding phase points in the volume element $dq\,dp$ around the point (q,p) in phase space is given by

$$dw = \rho(q,p,t)\, dq\, dp$$

So, the ensemble average $\langle f \rangle$ of a physical quantity $f(q,p)$ is defined as

$$\langle f \rangle = \int f(q,p)\, \rho(q,p,t)\, dq\, dp$$

Special case

when $\frac{\partial \rho}{\partial t} = 0$

then the ensemble average $\langle f \rangle$ is independent of time. Such ensemble is said to be stationary.